Unit Title: Power to the Variable

INSTRUCTIONAL UNIT AUTHORS

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BASED ON A CURRICULUM OVERVIEW SAMPLE AUTHORED BY

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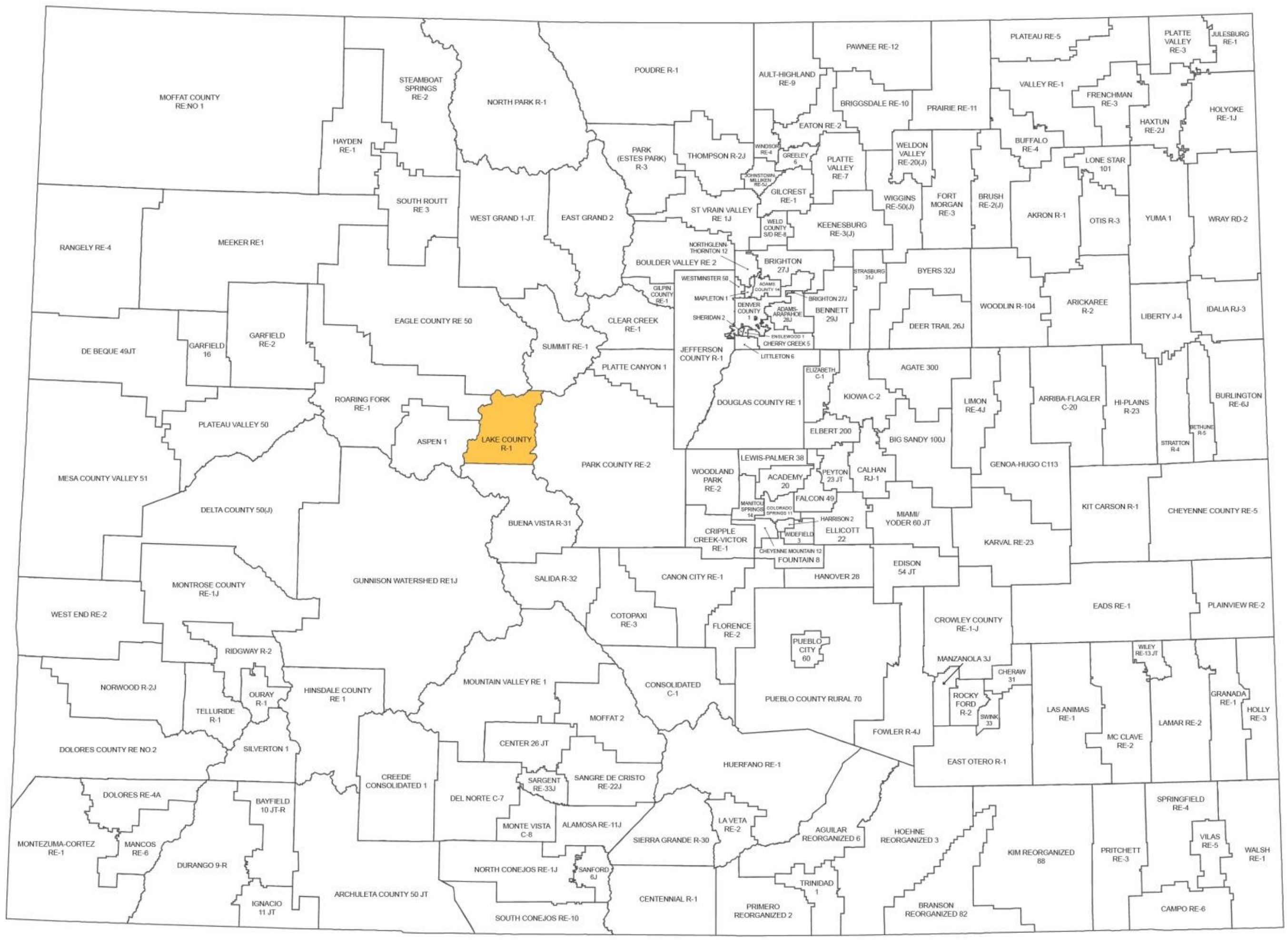

Colorado's District Sample Curriculum Project

This unit was authored by a team of Colorado educators. The template provided one example of unit design that enabled teacher-authors to organize possible learning experiences, resources, differentiation, and assessments. The unit is intended to support teachers, schools, and districts as they make their own local decisions around the best instructional plans and practices for all students.

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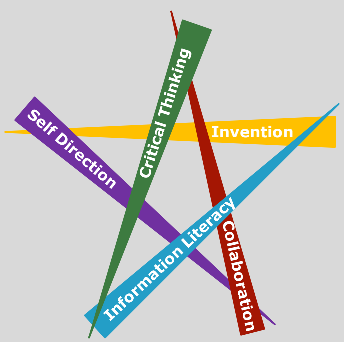
Colorado Teacher-Authored Sample Instructional Unit

Content Area	Mathematics	**Grade Level**	High School
Course Name/Course Code	Algebra I		

Standard	**Grade Level Expectations (GLE)**	**GLE Code**
1. Number Sense, Properties, and Operations	1. The complex number system includes real numbers and imaginary numbers	MA10-GR.HS-S.1-GLE.1
	2. Quantitative reasoning is used to make sense of quantities and their relationships in problem situations	MA10-GR.HS-S.1-GLE.2
2. Patterns, Functions, and Algebraic Structures	1. Functions model situations where one quantity determines another and can be represented algebraically, graphically, and using tables	MA10-GR.HS-S.2-GLE.1
	2. Quantitative relationships in the real world can be modeled and solved using functions	MA10-GR.HS-S.2-GLE.2
	3. Expressions can be represented in multiple, equivalent forms	MA10-GR.HS-S.2-GLE.3
	4. Solutions to equations, inequalities and systems of equations are found using a variety of tools	MA10-GR.HS-S.2-GLE.4
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics condense the information in data sets into usable knowledge	MA10-GR.HS-S.3-GLE.1
	2. Statistical methods take variability into account supporting informed decisions making through quantitative studies designed to answer specific questions	MA10-GR.HS-S.3-GLE.2
	3. Probability models outcomes for situations in which there is inherent randomness	MA10-GR.HS-S.3-GLE.3
4. Shape, Dimension, and Geometric Relationships	1. Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically	MA10-GR.HS-S.4-GLE.1
	2. Concepts of similarity are foundational to geometry and its applications	MA10-GR.HS-S.4-GLE.2
	3. Objects in the plane can be described and analyzed algebraically	MA10-GR.HS-S.4-GLE.3
	4. Attributes of two- and three-dimensional objects are measurable and can be quantified	MA10-GR.HS-S.4-GLE.4
	5. Objects in the real world can be modeled using geometric concepts	MA10-GR.HS-S.4-GLE.5

Colorado 21st Century Skills



Critical Thinking and Reasoning: *Thinking Deeply, Thinking Differently*

Information Literacy: *Untangling the Web*

Collaboration: *Working Together, Learning Together*

Self-Direction: *Own Your Learning*

Invention: *Creating Solutions*

Mathematical Practices:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Titles	**Length of Unit/Contact Hours**	**Unit Number/Sequence**
Power to the Variable	6 weeks	2

Unit Title	Power to the Variable		**Length of Unit**	6 weeks
Focusing Lens(es)	Modeling	**Standards and Grade Level Expectations Addressed in this Unit**	MA10-GR.HS-S.1-GLE.2 MA10-GR.HS-S.2-GLE.2 MA10-GR.HS-S.2-GLE.3 MA10-GR.HS-S.2-GLE.4	
Inquiry Questions (Engaging-Debatable):	• What are the parameters that affect gas mileage in a car and how would you model them? (MA10-GR.HS-S.2-GLE.2-EO.b.i) • What are the consequences of a population that grows exponentially?			
Unit Strands	Algebra: Reasoning with Equations and Inequalities Algebra: Creating Equations Algebra: Seeing Structure in Expressions Functions: Interpreting Functions Functions: Linear and Exponential Models			
Concepts	Models, quantity, growth, decay, constant rate of change, constant rate of growth, functions, linear functions, exponential functions, exponentially, linearly, quadratically, polynomial function, arithmetic sequence, geometric sequence, relationships, tables, graphs, equations, parameters, equations, inequalities, real world contexts			

Generalizations **My students will Understand that…**	**Guiding Questions** **Factual**	**Conceptual**
Linear and exponential functions provide the means to model constant rates of change and constant rates of growth, respectively (MA10-GR.HS-S.2-GLE.2-EO.a)	How do you determine from an equation whether an exponential function models growth or decay? How do you determine whether a situation can be modeled by a linear function, an exponential function, or neither? What are typical situations modeled by linear functions? What are typical situations modeled by exponential functions?	Why are differences between linear and exponential functions visible in equations, tables and graphs? Why does a common difference indicate a linear function and common ratio an exponential function?
A quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. (MA10-GR.HS-S.2-GLE.3-EO.a.iii)	How does the rate of growth in linear and exponential functions differ? How can you determine when an exponential function will exceed a linear function?	Why can so many situations be modeled by exponential growth? Why is important to consider the limitations of an exponential model?

Linear and exponential functions model arithmetic and geometric sequences respectively. (MA10-GR.HS-S.2-GLE.2-EO.a.ii)	How can you determine the slope and y-intercept of an arithmetic sequence? How can you determine the ratio for a geometric sequence? How do you know whether a sequence is arithmetic or geometric?	Why do linear and exponential functions model so many situations? Why is the domain of a sequence a subset of the integers?
The interpretation of the parameters of equations and inequalities must consider real world contexts. (MA10-GR.HS-S.2-GLE.2-EO.b.i)	What is a coefficient? How do you choose coefficients given a set of data?	Why are coefficients sometimes represented with letters? Why does changing coefficients affect a model? Why would you model a context with an inequality rather than an equation?
The generation of equivalent exponential functions by applying properties of exponents sheds light on a problem context and the relationships between. (MA10-GR.HS-S.2-GLE.3-EO.b.i.3)	How do properties of exponents simplify exponential expressions?	Why does a number raised to the power of zero equal one? Why do exponential patterns explain negative exponents?

Key Knowledge and Skills: **My students will…**	*What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.*

- Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (MA10-GR.HS-S.1-GLE.2-EOa.i.1,2)
- Use the properties of exponents to transform expressions for exponential functions with integer exponents. (MA10-GR.HS-S.2-GLE.3-EO.b.i.3)
- Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (MA10-GR.HS-S.2-GLE.4-EO.c.i) (The authors of this instructional unit decided to move this skill to the "All Systems Go" unit.)
- Create equations and inequalities in one variable and use them to solve problems; include equations arising from linear, quadratic, and exponential function with integer exponents. (MA10-GR.HS-S.2-GLE.4-EO.a.i)
- Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (MA10-GR.HS-S.2-GLE.2-EO.a.i.1)
- Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. (MA10-GR.HS-S.2-GLE.2-EO.a.i.2)
- Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (MA10-GR.HS-S.2-GLE.2-EO.a.i.3)
- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (MA10-GR.HS-S.2-GLE.2-EO.a.ii)
- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (MA10-GR.HS-S.2-GLE.2-EO.a.iii)
- Interpret the parameters in a linear or exponential (domain of integers) function in terms of a real world context and prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. (MA10-GR.HS-S.2-GLE.2-EO.b.i)

Critical Language: includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline. EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: *"Mark Twain exposes the hypocrisy of slavery through the use of satire."*	
A student in ______________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):	*In a linear function, as the coefficient of x increases, the slope gets steeper.* *Exponential functions model situations where a quantity has a constant rate of growth, such as doubling every year.*
Academic Vocabulary:	Transform, model, create, interpret, situations, real world contexts, growth, decay, relationships, tables, graphs,
Technical Vocabulary:	Quantity, constant rate of change, constant rate of growth, functions, linear functions, exponential functions, exponentially, linearly, quadratically, polynomial function, arithmetic sequence, geometric sequence, equations, parameters, equations, inequalities, common difference, common ratio, properties, parameter, coefficient,

Unit Description:	This unit focuses on a formal introduction to exponential functions. The students start with exploring exponential growth through geometric sequences that either grow or decay. As the students learn about geometric sequences, they continually compare them to arithmetic sequences, building to linear and exponential functions. Student fluency with these functions improves through multiple experiences with tables, graphs, equations and contexts. Then students examine the differences in the growth rates of linear, exponential, and polynomial functions leading to a formal proof of how linear functions grow by constant differences and exponential functions grow by common factors.
Unit Generalizations	
Key Generalization:	Linear and exponential functions provide the means to model constant rates of change and constant rates of growth, respectively
Supporting Generalizations:	Linear and exponential functions model arithmetic and geometric sequences respectively
	The interpretation of the parameters of equations and inequalities must consider real world contexts
	A quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically
	The generation of equivalent exponential functions by applying properties of exponents sheds light on a problem context and the relationships between

Performance Assessment: *The capstone/summative assessment for this unit.*	
Claims: (Key generalization(s) to be mastered and demonstrated through the capstone assessment.)	Linear and exponential functions provide the means to model constant rates of change and constant rates of growth, respectively.
Stimulus Material: (Engaging scenario that includes role, audience, goal/outcome and explicitly connects the key generalization)	You are a scientist who works at Mauna Loa observatory in Hawaii who measures CO_2 concentration in the atmosphere. You have data from the last 50 years (http://www.esrl.noaa.gov/gmd/ccgg/trends/#mlo_full). You are presenting to the governor about your data, including a prediction about what the CO_2 level will be in Hawaii in the year 2050. In order to create your prediction you will need to determine if it should be modeled by a linear or exponential function based on the rate of growth. [Note: A teacher may choose to pick another set of data to work with if it will be more engaging or relevant to their students]
Product/Evidence: (Expected product from students)	Students will model the data with both a linear function and an exponential function and use it to predict the CO2 concentration in the year 2050 and also when the CO_2 concentration will be above 500 parts per million. Students will discuss and document which model makes a better prediction and the limitations of each model. Students will be expected to show evidence of each step of the modeling process: • Problem – Show an understanding of what is being asked and what they are modeling. • Formulate – Create a linear and exponential function for the data showing each model in an equation, graph and table. • Compute – Calculate the CO_2 levels for 2050 and the date at which the CO_2 will be above 500 parts per million with each model. • Interpret – Interpret the parameters of the functions in the context of CO_2 concentration over time. • Validate – Check the model for accuracy by predicting the CO_2 concentration for intermediate years to assess the reasonableness of each model. Students will have the following products for their report:

	• Provide a written report of each part of the modeling process described above and a statement of final conclusions including the limitations of each model. • Create a 2 to 3 minute digital story (video) that the governor would be able to watch and understand the prediction and the limitations of the prediction.
Differentiation: (Multiple modes for student expression)	Students can write their report using a template showing each step of the modeling process and sentence starters to scaffold their writing. (http://www.mathsisfun.com/algebra/mathematical-models.html, http://caccssm.cmpso.org/high-school-modeling-task-force) Students can write their report in the form of a journal article and consider submitting it for publication.

Texts for independent reading or for class read aloud to support the content	
Informational/Non-Fiction	**Fiction**
	One Grain of Rice: A Mathematical Folktale by Demi (Lexile level 830) *The King's Chessboard* by David Birch (Lexile level 270+) *Anno's Magic Seeds* by Mitsumasa Anno (Lexile level 270+)

Ongoing Discipline-Specific Learning Experiences				
1.	Description:	Think/work like a mathematician – Expressing mathematical reasoning by constructing viable arguments, critiquing the reasoning of others. [Mathematical Practice 3]	Teacher Resources:	http://www.insidemathematics.org/index.php/standard-3 (examples of constructing viable arguments) http://quizlet.com/22134361/cpm-index-cards-of-teaching-strategies-flash-cards/ (teaching strategies to encourage class discussions)
			Student Resources:	N/A
	Skills:	Provide justification for arguments through a series of logical steps while using correct mathematical vocabulary. Analyze and critique the arguments of other students	Assessment:	Students justify their choice of a mathematical model (linear or exponential). Students can also critique the reasoning of a fellow student. Students will need to be precise with their language such as the use linear, arithmetic, exponential, geometric, rate of growth and common difference.
2.	Description:	Think/work like a mathematician – Engaging in the practice of modeling the solution to real world problems [Mathematical Practice 4]	Teacher Resources:	http://www.corestandards.org/Math/Content/HSM (common core state standards description of the modeling process) http://blog.mrmeyer.com/?p=16301 (Dan Meyer discussion on modeling) http://threeacts.mrmeyer.com (examples of 3-act problems)
			Student Resources:	N/A

	Skills:	Model real world problems mapping relationships with appropriate models of functions, analyze relationships to draw conclusions, interpret results in relation to context, justify and defend the model, and reflect on whether results make sense	Assessment:	Modeling Problems Students use linear and exponential models to analyze relationships presented in equations, graphs, tables and contexts and interpret the model in relation to the context to determine if the model makes sense.
3.	Description:	Mathematicians are fluent with analytic geometry of lines	Teacher Resources:	https://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/graphing-slope-intercept/e/graphing_linear_equations (videos and practice with linear functions)
			Student Resources:	http://www.thatquiz.org/tq-0/math/algebra/ (linear equation practice)
	Skills:	Write linear equations from two points or from a point and a slope	Assessment:	Fluency Problems Students build fluency with linear functions through consistent practice with building and analyzing linear equations, tables, graphs and contexts.

Prior Knowledge and Experiences

Student familiarity with manipulating and graphing linear equations will provide a strong foundation for the linear modeling in this unit. It is also helpful if students have a degree of comfort with properties of exponents to support their work with exponential functions.

Learning Experience # 1

The teacher may provide a context of simple geometric growth (e.g., a rabbit population or cell growth) that doubles or triples, so that students can explore how the pattern grows, recognize the growth is nonlinear and represent the growth using a model.

Enactive: Students can use manipulatives to represent the population at early stages in the growth process.
Iconic: Students can represent the size of population in a table and graph for several stages in the growth process and predict the size of the population for a future stage.
Symbolic: Students can represent the growth of the population with an exponential equation.

Teacher Notes:	Students should find the manipulatives to be inefficient relatively quickly but it is important for them to experience using this representation to reinforce how exponential contexts grow differently than linear contexts. Students might try to use their knowledge of linear models when using manipulatives, tables, graphs, and equations. It is important for students to discuss these naïve conceptions of linearity and to analyze why a linear model is inadequate.
Generalization Connection(s):	Linear and exponential functions model arithmetic and geometric sequences respectively

<table>
<tr><td>Teacher Resources:</td><td colspan="2">http://map.mathshell.org/materials/tasks.php?taskid=268&subpage=apprentice (a performance task on cells which double)
http://www.mathematicsvisionproject.org/uploads/1/1/6/3/11636986/sec1_mod3_linexpfunctions_se_901412.pdf (unit on linear and exponential functions)</td></tr>
<tr><td>Student Resources:</td><td colspan="2">One Grain of Rice: A Mathematical Folktale by Demi
http://www.otherwise.com/population/exponent.html (an applet that is a visual for simple exponential growth)</td></tr>
<tr><td>Assessment:</td><td colspan="2">Students mastering the concept and skills of this lesson should be able to answer questions such as:
How can we tell when a sequence is growing in a pattern that is not linear by a table, graph or equation?
How can you find the equation for a geometric growth context?
What are examples of geometric growth from the real world?</td></tr>
<tr><td rowspan="2">Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>http://www.doe.virginia.gov/instruction/mathematics/resources/vocab_cards/index.shtml (examples of visual vocabulary)
http://www.graniteschools.org/depart/teachinglearning/curriculuminstruction/math/Pages/MathematicsVocabulary.aspx (examples of secondary vocabulary cards)
http://www.virtualnerd.com/algebra-1/quadratic-equations-functions/linear-exponential-comparison/linear-exponential-comparison-graphing-examples/determine-function-type-from-graph (examples of a linear and nonlinear graphs and equations)</td><td>Students can create vocabulary cards and use them when responding to questions
Students can orally explain the differences between a linear and non-linear pattern in a table, graph, and equation</td></tr>
<tr><td rowspan="2">Extensions for depth and complexity:</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>http://math.about.com/od/exponentialdecay/a/Exponential-Growth-3.htm (example of real world exponential growth)</td><td>Students can present independent research on additional examples of exponential functions/geometric series</td></tr>
<tr><td>Key Knowledge and Skills:</td><td colspan="2">• Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table)</td></tr>
<tr><td>Critical Language:</td><td colspan="2">Model, growth, equation, table, graph, linear, exponential, rate of change, term, geometric growth, sequence, arithmetic sequence, geometric sequence, consecutive, stages, doubles, triples
The growth of this population in not linear because the rate of change is not constant.
The growth of this population is exponential because to find the next term in the sequence I multiply the last term by the same number each time.</td></tr>
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<table>
<tr><td colspan="3">Learning Experience # 2</td></tr>
<tr><td colspan="3">The teacher may provide a situation of exponential decay so that students can explore how to model a decaying geometric sequence.
Enactive: Students can observe, measure, and collect the data.
Iconic: Students can represent their data in the form of a table and graph.
Symbolic: Students can represent their data in the form of an equation.</td></tr>
<tr><td>Teacher Notes:</td><td colspan="2">One example of exponential decay is to have students measure the height of a ball every time it bounces after being dropped from a height of 2 meters. Another example could be to have students look up blue book values of a car over time. Students are expanding their learning from the first learning experience by being exposed to a decay rather than growth scenario and the data will not be perfectly modeled by an equation. Students will need guidance in finding the ratio between consecutive values in the sequence when dealing with data that is not precisely exponential.</td></tr>
<tr><td>Generalization Connection(s):</td><td colspan="2">Linear and exponential functions model arithmetic and geometric sequences respectively</td></tr>
<tr><td>Teacher Resources:</td><td colspan="2">http://www.kbb.com/whats-my-car-worth/ (data for a particular make and model's value)
http://www.illustrativemathematics.org/illustrations/347 (basketball rebounds)
http://ib-physics-sl.blair.pasadenausd.org/modules/locker/files/get_group_file.phtml?gid=1399819&fid=9225288 ("messy" ball bounce data—see page 4)</td></tr>
<tr><td>Student Resources:</td><td colspan="2">www.desmos.com (online graphing calculator to plot data and assess fit on an equation)</td></tr>
<tr><td>Assessment:</td><td colspan="2">Students mastering the concept and skills of this lesson should be able to answer questions such as:
How is the ratio between two terms in a geometric sequence found?
What are two different ways that a set of data can decrease? (e.g., linear and exponential)
What real world examples of linear and exponential decay? How do the contexts differ?
How can you determine from an equation, table, and graph if an exponential function represents decay or growth?</td></tr>
<tr><td rowspan="2">Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>http://www.encyclopedia.com/doc/1G2-3407500040.html (premade table of "clean" data for an exponential rate of decay)
http://www.math-aids.com/Graph_Paper/ (graph template)</td><td>Students can create an equation by analyzing data in a table showing a precise or clean exponential decay model.
Students can create a visual such as a graph comparing linear and exponential decay</td></tr>
<tr><td rowspan="2">Extensions for depth and complexity:</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>http://education.ti.com/~/media/61029F0123F4416E8EB2173B61503A45 (example of an activity to explore asymptotic behavior for a basketball bouncing)</td><td>Students can create presentations that describe why exponential functions have asymptotic behavior (e.g., does the basketball bounce forever)?</td></tr>
<tr><td>Key Knowledge and Skills:</td><td colspan="2">• Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table)</td></tr>
</table>

Critical Language:	Model, table, graph, linear, exponential, rate of change, ratio, decay, term, geometric decay, sequence, geometric sequence, consecutive, stages *I can find the ratio of a geometric sequence by dividing any two consecutive terms of the sequence.*

<table>
<tr><td colspan="3">Learning Experience # 3</td></tr>
<tr><td colspan="3">The teacher may provide a variety of sequences in the form of tables, graphs, equations, and contexts so that students can distinguish between geometric and arithmetic sequences and connect them to multiple representations.
Iconic: Students can categorize sequences as either arithmetic or geometric when represented as tables, graphs, equations, and contexts.
Symbolic: Students can match equivalent symbolic expressions of a sequence.</td></tr>
<tr><td>Teacher Notes:</td><td colspan="2">This activity works best as a card sort, with students in groups of 2, 3, or 4. Students need to be required to justify how they sorted the cards, a card is only moved if they can justify their reasoning to a partner. Teachers should think carefully about whether to group heterogeneously or by ability. The teacher should lead a discussion at the end of the learning experience to help the students summarize the properties of arithmetic and geometric sequences onto a graphic organizer or note sheet.</td></tr>
<tr><td>Generalization Connection(s):</td><td colspan="2">Linear and exponential functions model arithmetic and geometric sequences respectively</td></tr>
<tr><td>Teacher Resources:</td><td colspan="2">http://www.teacherspayteachers.com/Product/Algebra-Linear-or-Exponential-829683 ($1.00 cost for a card sort with linear and exponential representations)</td></tr>
<tr><td>Student Resources:</td><td colspan="2">N/A</td></tr>
<tr><td>Assessment:</td><td colspan="2">Students mastering the concept and skills of this lesson should be able to answer questions such as:
What are the defining characteristics of geometric and arithmetic sequences?
Given a table, graph, or equation representing a sequence, how can the representation be used to create a different representation (i.e., table to graph)?</td></tr>
<tr><td rowspan="2">Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>https://commoncorealgebra1.wikispaces.hcpss.org/file/detail/F.LE.A.1b%20Lesson%20Constant%20Rate%20Exploration.doc (Frayer model templates at end of the lesson for linear and exponential functions)
Teachers may provide sentence starters:
I know this is a geometric sequence because . . .
I know this is linear sequence because . . .
This table/graph/equation/context matches this other table/graph/equation/context because . . .
http://www.curee-paccts.com/files/publication/[site-timestamp]/Teaching%20strategies%20to%20improve%20writing%20in%20mathematics.pdf (examples of sentence starters and how to use them in a math classroom)</td><td>Students can create and use a Frayer model for exponential and linear equations to support their justifications during the card sort
Students can justify if a representation is linear or exponential by using sentence starters</td></tr>
</table>

Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://www.theteachertoolkit.com/index.php/tool/card-sort (card sorting template)	Students can create their own card sort for exponential and linear functions
Key Knowledge and Skills:	• Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table)	
Critical Language:	Model, table, graph, linear, exponential, rate of change, constant rate of growth, constant difference, ratio, sequence, arithmetic, geometric sequence, decay, growth	

Learning Experience # 4	
The teacher may provide an example of two financial institutions, one that gives simple interest and one that gives compound interest, so that students can explore which type of function models these situations. *Enactive*: Students can act out a scenario for each bank. Using play money, one student can act as bank the other as the costumer and then switch roles. After acting out several years for each bank students can make a prediction about the best bank to invest their money and justify their choice to a partner. *Iconic*: Students can make a table representing how much money they would have with each bank after (e.g., 1 year, 2 years, 3 years…). Students can analyze their table and revisit their prediction about the best bank. *Symbolic*: Students can analyze the table and create an equation using function notation. Students can interpret the parameters in terms of the context and predict how much money will be in the account after 100 years. They also can determine how much money is in their account after .5 years, or 7.5 years. Other examples to model can then be provided (i.e., different interest rates or initial account values).	
Teacher Notes:	One goal of this activity is to formalize the transition from arithmetic and geometric sequences to linear and exponential functions. Thus it is important for students to use function notation when creating the equations for this learning experience. Another key focus of this activity is to have students study a variety of growth examples (e.g., 5% growth, 10% growth) instead of simply doubling or tripling. It is also important to have the students interpret the parameters of the functions within the context provided. After this learning experience, student could benefit from some skill practice in multiple representations of linear and exponential functions.
Generalization Connection(s):	Linear and exponential functions provide the means to model constant rates of change and constant rates of growth, respectively The interpretation of the parameters of equations and inequalities must consider real world contexts
Teacher Resources:	http://map.mathshell.org/materials/lessons.php?taskid=426&subpage=concept (card sort activity on simple versus compound interest)
Student Resources:	http://investor.gov/tools/calculators/compound-interest-calculator (compound interest calculator) http://www.bankrate.com/calculators/savings/compound-interest-calculator-tool.aspx (compound interest calculator)
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: How are functions different than sequences? How are arithmetic sequences and linear functions related? How are geometric sequences and exponential functions related? When is compound interest better than simple interest? What does each part of an equation for compound interest represent?

<table>
<tr><td></td><td colspan="2">How can you use a graph or table to determine the year an account will reach $1000?
How can you use a graph, table, or equation to determine the amount of money after 100 years?
Why can simple and compound interest be represented with a function?</td></tr>
<tr><td rowspan="2">Differentiation:
(Multiple means for students to access content and multiple modes for student to express understanding.)</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>http://map.mathshell.org/materials/lessons.php?taskid=426&subpage=concept (card sort activity)
https://www.youtube.com/watch?v=HaAJcWR5JtA (video showing simple and compound interest)
http://www.eisd.net/cms/lib04/TX01001208/Centricity/Domain/599/DoubleBubbleMap.pdf (thinking map for comparing and contrasting)</td><td>Students can sort fewer cards to focus first on recognizing the difference between simple and compound interest and connecting these ideas to previous learning experiences focused linear and exponential scenarios
Students can create a graphic organizer to comparing simple and compound interest</td></tr>
<tr><td rowspan="2">Extensions for depth and complexity:</td><td>Access (Resources and/or Process)</td><td>Expression (Products and/or Performance)</td></tr>
<tr><td>http://www.mathwarehouse.com/compound-interest/continuously-compounded-interest.php (continually compounded interest)
http://www.purplemath.com/modules/logs.htm (basics of logarithms)</td><td>Students can create an excel spreadsheet to calculate the interest on a savings account they researched at a local bank which offers continually compounded interest
Students can present research how logarithms provide a method for determining the exact number amount of time required to reach a set amount of money</td></tr>
<tr><td>Key Knowledge and Skills:</td><td colspan="2">• Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table)
• Interpret the parameters in a linear or exponential (domain of integers) function in terms of a real world context
• Recognize situations in which one quantity changes at a constant rate per unit interval relative to another
• Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another
• Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays</td></tr>
<tr><td>Critical Language:</td><td colspan="2">Model, table, graph, equation, growth, percent growth, linear, exponential, function, rate of change, constant rate of growth, constant difference, ratio, sequence, arithmetic sequence, geometric sequence</td></tr>
</table>

Learning Experience # 5

The teacher may provide situations such as population growth or decline, so that students can investigate how parameters affect models for equations of the form, $y = mx+b$ and $y = ab^x$.

Iconic: Students can examine tables and graphs of data sets to determine if a linear or exponential function would best model the situation. Students can use the modeling process throughout this learning experience, including the identification of variables, reasonableness of a model, and defining of appropriate domains and ranges (http://www.corestandards.org/Math/Content/HSM).

Symbolic: Students can create equations and manipulate parameters to match given situations (http://www.math.uri.edu/~pakula/expslide1.htm). Students can also compare two or more functions with different parameters to find the point of intersection and interpret this point.

Teacher Notes:	Both linear and exponential situations should be provided. To emphasize the importance of each parameter of a linear and exponential function, many different variations with different starting points and growth rates should be provided. Real-world data can be used so students can determine an informal curve of best fit and parameters can be interpreted. In order to explore micro-models, students should be asked about a reasonable domain for the real world equations.	
Generalization Connection(s):	Linear and exponential functions provide the means to model constant rates of change and constant rates of growth, respectively The interpretation of the parameters of equations and inequalities must consider real world contexts	
Teacher Resources:	www.desmos.com (graphing tool where parameters can be manipulated using sliders) http://finance.yahoo.com/ (historical stock prices) http://wwf.panda.org/what_we_do/endangered_species/rhinoceros/african_rhinos/black_rhinoceros/ (rhino population data) http://www.corestandards.org/Math/Content/HSM (description of modeling process)	
Student Resources:	http://www.math.uri.edu/~pakula/expslide1.htm (applet with sliders for exponential functions)	
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: Why does a model not always give a reasonable prediction? In the equation $y = ab^x$, explain the impact of changing the parameters *a* and *b*? In the equation y = mx+b, explain the impact of changing the parameters *m* and *b*? If the parameters are changed, what will happen to the output of a model? How do you explain the meaning of a parameter in terms of the context?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://learni.st/users/171/boards/2037-identifying-linear-and-exponential-models-common-core-standard-f-le-1 (video explaining how to identify exponential and linear functions from tables) http://www.ixl.com/math/algebra-1/identify-linear-quadratic-and-exponential-functions-from-graphs (graphs showing quadratic, exponential and linear functions) http://www.mathematicsvisionproject.org/uploads/1/1/6/3/11636986/sec1_mod4_lef_se_71713.pdf (example contexts with clear exponential or linear data)	Students can orally explain how to identify if a function is exponential or linear (video) Students can practice recognizing graphs of linear, exponential and quadratic functions Students can create models from “clean” data, which are clearly exponential or linear
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	www.desmos.com (graphing tool where parameters can be manipulated use sliders)	Students can explain the impact of changing the parameters *a, b, c* and *d* on the graph of the more complex exponential function $y=ab^{x-c}+d$
Key Knowledge and Skills:	• Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table) • Interpret the parameters in a linear or exponential (domain of integers) function in terms of a real world context • Recognize situations in which one quantity changes at a constant rate per unit interval relative to another	

	• Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another • Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays • Create equations and inequalities in one variable and use them to solve problems; include equations arising from linear, quadratic, and exponential function with integer exponents
Critical Language:	Model, table, graph, equation, growth, linear, exponential, function, rate of change, constant rate of growth, constant difference, ratio, sequence, arithmetic sequence, geometric sequence, domain, range, extrapolate, micro-model, parameter, limitation, variable

Learning Experience # 6

The teacher may provide examples of linear, quadratic and exponential growth so that student can explore and observe the differences in growth for exponential functions versus polynomial functions.

Iconic/Symbolic: Students can use a graphing tool to graph linear, quadratic and exponential functions and use the zoom feature to observe how the exponential function eventually exceeds the linear and quadratic. As students come to conclusions about which grows faster, they can justify their conclusions to other students. After exploring these three types of functions, students should move to a more general comparison of any polynomial function (e.g., $y = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x^1 + a_0$) to an exponential function.

Teacher Notes:	Students should be given ample time to explore the functions before justifying their conclusions. Students will likely need to be reminded to change the window size of their graph (i.e, zoom out) in order to examine what is happening with the functions. This learning experience is designed to provide an informal understanding of growth leading to a more formal proof comparing the growth of linear and exponential functions in the next learning experience.	
Generalization Connection(s):	A quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically	
Teacher Resources:	http://www.illustrativemathematics.org/illustrations/366 (exponential vs. linear growth) http://www.illustrativemathematics.org/illustrations/368 (exponential vs. linear growth 2) http://www.illustrativemathematics.org/illustrations/367 (exponential vs. polynomial growth) http://www.illustrativemathematics.org/illustrations/645 (compare a population growing exponentially to resources growing linearly)	
Student Resources:	www.desmos.com (graphing tool)	
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: Why are linear and quadratic functions both examples of polynomial functions? Which type of function, exponential or polynomial, grows the fastest? How would you demonstrate which function grows the quickest, quadratic or exponential?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://www.mathsisfun.com/algebra/linear-equations.html (review the form of a linear function) http://www.mathsisfun.com/algebra/quadratic-equation.html (review the form of a quadratic function)	Students can write linear equations prior to using the graphing tool to explore their growth Students can write quadratic equations prior to using the graphing tool to explore their growth

	http://www.mathsisfun.com/algebra/polynomials-general-form.html (review the form of polynomial function)	Students can write polynomial equations prior to using the graphing tool to explore their growth
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	www.desmos.com (graphing tool) http://mathonweb.com/help_ebook/html/functions_4.htm#linear (types of functions)	Students can demonstrate how exponential functions grow faster than any other type function not just polynomial functions
Key Knowledge and Skills:	• Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function • Create equations and inequalities in one variable and use them to solve problems; include equations arising from linear, quadratic, and exponential function with integer exponents	
Critical Language:	Function, quantity, increasing, linearly, exponentially, quadratically, linear, exponential, quadratic, polynomial function, intersection, rate of change, graph, table, domain, range, parameter	

Learning Experience # 7

The teacher may provide examples of linear and exponential functions so that students can explore rates of change (over equal intervals) for each type of function in order to develop a proof about the general equations, $y=ax+b$ or $y=ab^x$.

Iconic: Students can find and recognize the patterns in linear and exponential tables.

Symbolic: Students can prove using variable coefficients that linear functions grow by equal differences over equal intervals and then that exponential functions grow by equal factors over equal intervals.

Teacher Notes:	The teacher can determine the number of examples the class needs to examine to understand the pattern before moving to the abstract proof. It is important to highlight to students that an example is not a proof. The goal of a proof is to verify for *all* linear or exponential functions not just a few examples.
Generalization Connection(s):	Linear and exponential functions provide the means to model constant rates of change and constant rates of growth, respectively A quantity increasing exponentially exceeds a quantity increasingly linearly or quadratically
Teacher Resources:	http://www.illustrativemathematics.org/illustrations/350 (linear functions) http://www.illustrativemathematics.org/illustrations/362 (linear functions) http://www.illustrativemathematics.org/illustrations/363 (exponential functions) http://learnzillion.com/lessons/300-prove-that-linear-functions-grow-by-equal-differences-over-equal-intervals (video of a proof that linear functions grow by equal differences) http://learnzillion.com/lessons/301-prove-exponential-functions-grow-by-equal-factors-over-equal-intervals (video of a proof that exponential functions grow by equal factors)
Student Resources:	N/A
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: How can you prove that a linear function grows by equal differences over equal intervals? How can you prove that an exponential function grows by equal factors over equal intervals?

Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://learnzillion.com/lessons/300-prove-that-linear-functions-grow-by-equal-differences-over-equal-intervals (video of a proof that linear functions grow by equal differences) http://learnzillion.com/lessons/301-prove-exponential-functions-grow-by-equal-factors-over-equal-intervals (video of a proof that exponential functions grow by equal factors)	Students can explain proofs in their own words after watching videos of the proofs
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://education.ti.com/en/us/activity/detail?id=F4CDF65806CA40BD8458D2420E667D2D (activity comparing exponential and power functions)	Students can provide proof that exponential functions grow faster than power functions
Key Knowledge and Skills:	• Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals • Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters	
Critical Language:	Prove, linear function, exponential function, equal differences, equal factors, equal intervals	

Learning Experience # 8	
The teacher may provide an example of a population that doubles every month so that students can practice rewriting exponential functions using properties of exponents. *Symbolic*: Students are provided with exponential functions or expressions and asked to transform them using the properties of exponents. Students can then explain the meaning of the new base and exponent in the context.	
Teacher Notes:	An example of a way to rewrite the expression might be to ask students to write the expression with the independent variable being both months and then rewrite it as years. Note that this standard is tested both in Algebra I and Algebra II. In Algebra I, the examples are limited to integer exponents. The focus of this learning experience is on seeing structure in expressions (Mathematical Practice 7: Look For and Make Use of Structure). It is valuable to review the properties of exponents with students before tackling this activity.
Generalization Connection(s):	The generation of equivalent exponential functions by applying properties of exponents sheds light on a problem context and the relationships between
Teacher Resources:	http://www.shmoop.com/common-core-standards/ccss-hs-a-sse-3c.html (detailed explanation using properties to transform exponential functions)
Student Resources:	N/A

Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: How can exponential expressions be rewritten? Can you explain why $10(2^{-x})$ is equivalent to $10(.5^{x})$?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide a handout with the properties of exponents http://hotmath.com/hotmath_help/topics/properties-of-exponents.html (properties of exponents) http://www.mathwarehouse.com/algebra/exponents/solve-exponential-equations-how-to.php (explanations and examples of how to solve exponential equations)	Students can practice rewriting exponential expressions prior to this learning experience
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://hotmath.com/hotmath_help/topics/properties-of-exponents.html (properties of exponents)	Student can create symbolic proofs for each of the properties of exponents
Key Knowledge and Skills:	• Use the properties of exponents to transform expressions for exponential functions with integer exponents	
Critical Language:	Exponential, equivalent forms, seeing structure, expression, base.	