Colorado’s District Sample Curriculum Project

Colorado Teacher-Authored Instructional Unit Sample

Mathematics
8th Grade

Unit Title: Walk the Line

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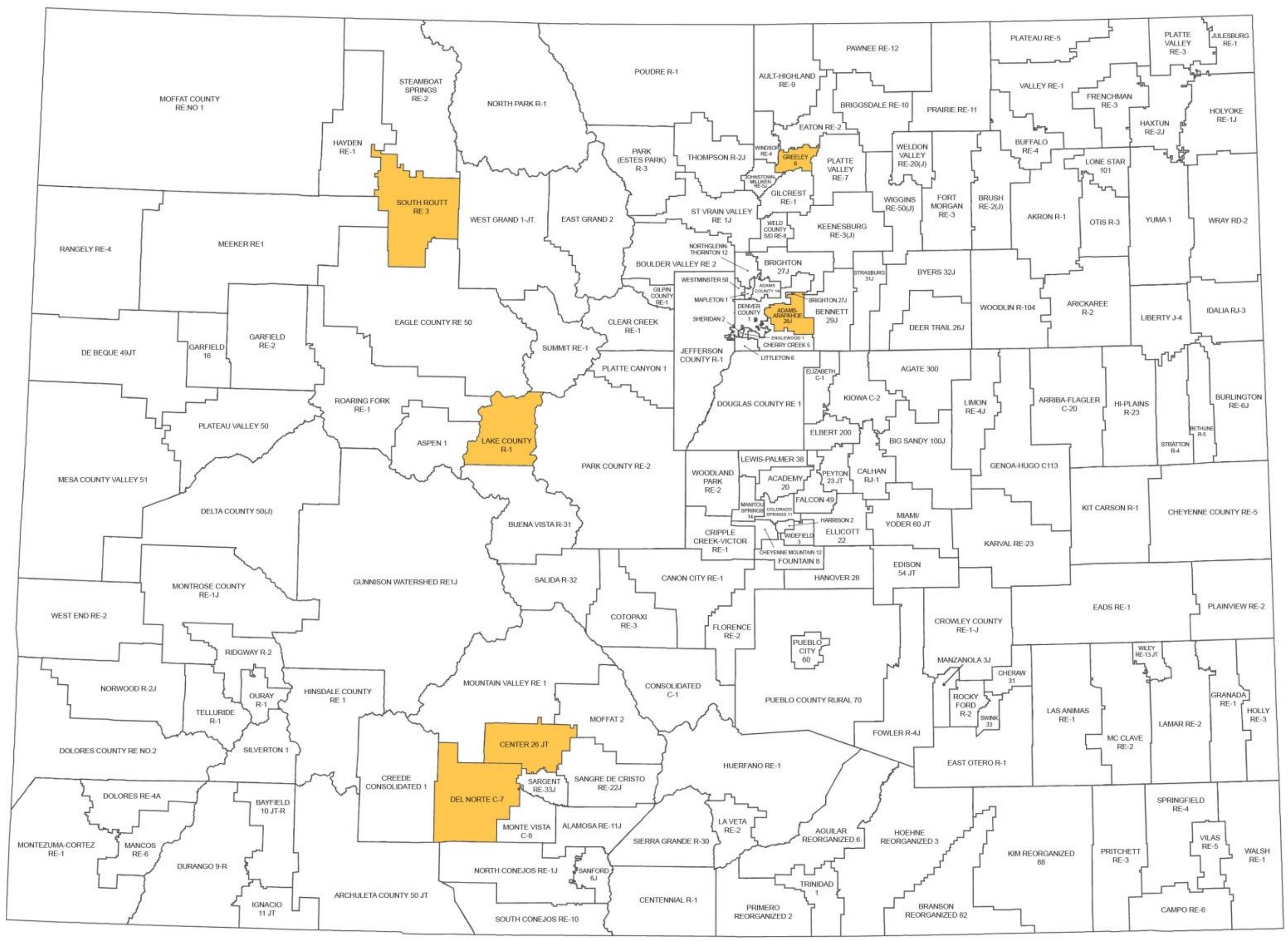

This unit was authored by a team of Colorado educators. The template provided one example of unit design that enabled teacher-authors to organize possible learning experiences, resources, differentiation, and assessments. The unit is intended to support teachers, schools, and districts as they make their own local decisions around the best instructional plans and practices for all students.

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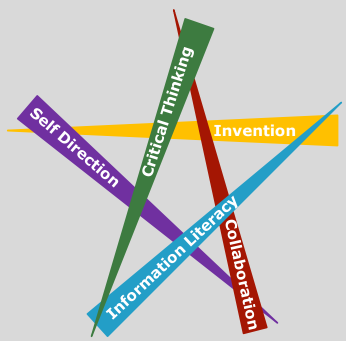
Colorado Teacher-Authored Sample Instructional Unit

Content Area	Mathematics	**Grade Level**	8^{th} Grade
Course Name/Course Code			

Standard	**Grade Level Expectations (GLE)**	**GLE Code**
1. Number Sense, Properties, and Operations	1. In the real number system, rational and irrational numbers are in one to one correspondence to points on the number line	MA10-GR.8-S.1-GLE.1
2. Patterns, Functions, and Algebraic Structures	1. Linear functions model situations with a constant rate of change and can be represented numerically, algebraically, and graphically	MA10-GR.8-S.2-GLE.1
	2. Properties of algebra and equality are used to solve linear equations and systems of equations	MA10-GR.8-S.2-GLE.2
	3. Graphs, tables and equations can be used to distinguish between linear and nonlinear functions	MA10-GR.8-S.2-GLE.3
3. Data Analysis, Statistics, and Probability	1. Visual displays and summary statistics of two-variable data condense the information in data sets into usable knowledge	MA10-GR.8-S.3-GLE.1
4. Shape, Dimension, and Geometric Relationships	1. Transformations of objects can be used to define the concepts of congruence and similarity	MA10-GR.8-S.4-GLE.1
	2. Direct and indirect measurement can be used to describe and make comparisons	MA10-GR.8-S.4-GLE.2

Colorado 21^{st} Century Skills



Critical Thinking and Reasoning: *Thinking Deeply, Thinking Differently*

Information Literacy: *Untangling the Web*

Collaboration: *Working Together, Learning Together*

Self-Direction: *Own Your Learning*

Invention: *Creating Solutions*

Mathematical Practices:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Unit Titles	**Length of Unit/Contact Hours**	**Unit Number/Sequence**
Walk the Line	6 weeks	4

Unit Title	Walk the Line	**Length of Unit**	6 weeks
Focusing Lens(es)	Change	**Standards and Grade Level Expectations Addressed in this Unit**	MA10-GR.8-S.2-GLE.1 MA10-GR.8-S.2-GLE.2 MA10-GR.8-S.2-GLE.3 MA10-GR.8-S.3-GLE.1
Inquiry Questions (Engaging-Debatable):	• Why do some problems have no solution? (MA10-GR8-S.2-GLE.3)		
Unit Strands	Expressions and Equations, Functions, Statistics and Probability		
Concepts	Linear equations, model, linear relationships, proportional relationships, rate of change, representations, right triangles, similar triangles, constant of proportionality, equivalence, slope, algebraic equations, one variable linear equations, intercept		

Generalizations **My students will Understand that…**	**Guiding Questions** **Factual**	**Conceptual**
Linear equations use $y = mx$ to model proportional relationships and $y = mx + b$ to model relationships in terms of a rate of change and initial value (MA10-GR.8-S.2-GLE.1-EO.a, e)	How do you know when the equation of a line will be in the form $y = mx$ verses $y = mx + b$? What properties of a function make it a linear function? (MA10-GR8-S.2-GLE.3-IQ.3) How would you determine if a function is not linear?	How does the rate of change relate to the slope of a line? How does changing the y-intercept value change the structure of your linear equation?
Rate of change and intercepts exist in all representations of linear relationships. (MA10-GR8-S.2-GLE.3-EO.b.i, b.ii, b.iii) and (MA10-GR.8-S.3-GLE.1-EO.d)	What are 3 ways to represent a linear relationship? What do m and b represent in a linear representation? How does the context of the linear relationship help to interpret the rate of change and initial value of linear function? How can the rate of change and initial value be determined from a graph, table and equation? How can you write a linear equation from a graph, table or context?	How do different representations of a linear relationship represent the same data? Why do we represent linear relationships using different representations? Why are patterns and relationships represented in multiple ways? (MA10-GR8-S.2-GLE.3-IQ.2)
Right triangles created between any two distinct points on a non-vertical line in a coordinate plane form similar triangle with a constant of proportionality equivalent to the slope of the line. (MA10-GR.8-S.2-GLE.1-EO.a, b)	What is slope? How can you calculate slope by creating a right triangle from any two distinct points on a non-vertical line?	Why is the slope equivalent to the scale factor used in similar triangles? Why are the legs of two similar triangles, created on the same non-vertical line, proportional?

The creation of equivalent algebraic equations provides the necessary foundation for solving linear equations in one variable. MA10-GR8-S.2-GLE.2-EO.a.i, a.ii)	What are examples of a linear one variable equation with no solutions? What are examples of linear one variable equation with infinite solutions? How does creating equivalent expressions lead to the solution of one variable linear equations?	Why can a one variable linear equation have no solutions or infinite solutions? How does the context of the problem affect the reasonableness of a solution? (MA10-GR8-S.2-GLE.2-IQ.3)

Key Knowledge and Skills: **My students will…**	*What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.*

- Graph proportional relationships, interpreting the unit rate as the slope of the graph (MA10-GR8-S.2-GLE.1-EO.a)
- Compare two different proportional relationships represented in different ways (MA10-GR8-S.2-GLE.1-EO.c)
- Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane (MA10-GR8-S.2-GLE.1-EO.d)
- Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. (MA10-GR8-S.2-GLE.1-EO.d)
- Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions; show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (MA10-GR8-S.2-GLE.2-EO.a.i)
- Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms (MA10-GR8-S.2-GLE.2-EO.a.ii)
- Construct a function to model a linear relationship between two quantities and determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph (MA10-GR8-S.2-GLE.3-EO.EO.b.i, b.ii)
- Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values (MA10-GR8-S.2-GLE.3-EO.b.iii)
- Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept (MA10-GR8-S.3-GLE.1-EO.d)

Critical Language: includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline.
EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: *"Mark Twain exposes the hypocrisy of slavery through the use of satire."*

A student in _______________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):	*There is a relationship between slope, unit rate, and rate of change in linear relationships.* *Slopes and initial values can be found for linear equations represented in different ways.*
Academic Vocabulary:	Calculate, interpret, graph, relationship, interpret, solve, construct, compare
Technical Vocabulary:	Linear equations, model, linear relationships, proportional relationships, rate of change, representations, right triangles, similar triangles, scale factor, equivalence, slope, algebraic equations, one variable linear equations, unit rate, intercept, origin, variable, solution, distributive property, like terms, coefficient, bivariate measurement data

Unit Description:	In Walk the Line, students explore representations of proportional and linear relationships. The authors of this unit begin by having students explore different representations of a proportional relationship and then move their understandings of proportional relationships to linear relationships. A major focus of this unit includes an investigation with slope and similar triangles. Lastly, students will be solving a variety of one-variable linear equations that have one, none, and infinitely many solutions.
Unit Generalizations	
Key Generalization:	Linear equations use $y = mx$ to model proportional relationships and $y = mx + b$ to model relationships in terms of a rate of change and initial value
Supporting Generalizations:	Right triangles created between any two distinct points on a non-vertical line in a coordinate plane form similar triangles with a constant of proportionality equivalent to the slope of the line
	Rate of change and intercepts exist in all representations of linear relationships
	The creation of equivalent algebraic equations provides the necessary foundation for solving linear equations in one variable

Performance Assessment: *The capstone/summative assessment for this unit.*	
Claims: (Key generalization(s) to be mastered and demonstrated through the capstone assessment.)	Linear equations use $y = mx$ to model proportional relationships and $y = mx + b$ to model relationships in terms of a rate of change and initial value (MA10-GR.8-S.2-GLE.1-EO.a, e)
Stimulus Material: (Engaging scenario that includes role, audience, goal/outcome and explicitly connects the key generalization)	A local charity has hired you to manage a fund raising event such as a walk-a-thon. The charity has determined the walk-a-thon will cost them $300 to host because of advertising and other miscellaneous costs. In order to participate, the charity has decided each volunteer must raise at least twenty dollars in pledges. The charity would like you to create a projected earnings report for the walk-a-thon. Your report must include a model of a linear relationship that indicates the earnings you hope to accrue and the calculations and equations that support these hopes.
Product/Evidence: (Expected product from students)	Students will create a projected earnings report for the charity. The earnings report should be based on a $300 initial start up cost and $20 in pledges per participant. The report should include: • a table showing the number of participants and project earnings, including the number of participants needed to break even (i.e., earn at least $300) • a graph displaying the number of participants and project earnings • an equation to determine the projected earnings given the number of participants • an explanation of the meaning of each part of the table, graph and equation, including the meaning of the x and y-intercepts and slope Students can also make recommendations in their report for modifying initial start-up costs or pledges per participant and their impact on the projected earnings.
Differentiation: (Multiple modes for student expression)	Students can be provided templates for creating the graph and table and sentence frames for their explanations (https://mathsentenceframes.wikispaces.com/). Students can compare different minimum pledge requirements for the charity. Students can use graphing software to create their report (http://illuminations.nctm.org/ActivityDetail.aspx?ID=204).

Texts for independent reading or for class read aloud to support the content	
Informational/Non-Fiction	**Fiction**
Roll, Slope, and Slide: A book about ramps by Michael (Lexile level 860)	N/A

Ongoing Discipline-Specific Learning Experiences				
1.	Description:	Think/work like a mathematician – Expressing mathematical reasoning by constructing viable arguments, **critiquing the reasoning of others** [Mathematical Practice 3]	Teacher Resources:	http://schools.nyc.gov/Academics/CommonCoreLibrary/TasksUnitsStudentWork/default.htm (lesson plans contains exemplars that could be replicated for students to critique the reasoning of others) http://map.mathshell.org/materials/index.php (samples and examples of student work to critique the validity of others) www.exemplars.com/resources/rubrics/assessment-rubrics (standards-based math rubric for the students to assess other's work)
			Student Resources:	N/A
	Skills:	Construct and communicate a complete and concise response, justify a conclusion using correct vocabulary, interpret and critique the validity of other's conclusions and reasoning, and identify errors and present correct solutions	Assessment:	Students analyze and defend their solutions for each major learning experience. Careful attention should be paid to precise use of vocabulary and symbols. Periodically throughout the unit, students can also be provided with flawed solutions and asked to identify, describe, and correct the flaw.
2.	Description:	Think/work like a mathematician – Engaging in the practice of modeling the solution to **real world problems** [Mathematical Practice 4]	Teacher Resources:	https://www.sites.google.com/a/cmpso.org/caccss-resources/k-8-modeling-task-force/k-8-modeling-resources (examples of modeling problems and resources for teachers on teaching and scoring them) http://www.insidemathematics.org/index.php/standard-4 (video examples of students modeling with mathematics) http://learnzillion.com/lessons/1722-solve-multistep-word-problems-using-model-drawing (video about modeling)http://blog.mrmeyer.com/?p=16301 (Dan Meyer discussion on modeling) http://threeacts.mrmeyer.com (examples of 3-act problems)
			Student Resources:	N/A

	Skills:	Devise a plan to apply mathematics to solve everyday problems by using and making stated assumptions and approximations to simplify a real-world situation. Map relationships between important quantities by selecting appropriate tools to create models. Analyze mathematical relationships between important quantities to draw conclusions. Determine if the results make sense. If necessary, change the model.	Assessment:	Modeling Problems Students utilize visual models for linear equations, such as graphs and tables to represent and analyze relationships of real-world problems to draw conclusions and interpret results in relation to the context of the problem.
3.	Description:	Mathematicians are fluent with solving one-variable linear equations	Teacher Resources:	http://www.mathedpage.org/attc/cover-up.pdf (describes a pre-formal method for solving linear equations called the cover-up method) http://www.purplemath.com/modules/solvelin.htm (describes the formal method for solving linear equations)
			Student Resources:	http://www.fi.uu.nl/wisweb/en/ (applets called solving equations or linear equations are designed to practice solving one variable equations) http://www.ixl.com/math/grade-8 (practice questions for single variable equations)
	Skills:	Solve one-variable linear equations	Assessment:	Fluency Problems Students build fluency with solving one variable linear equations, including cases with infinitely many solutions or no solutions as well as cases requiring algebraic manipulation through consistent practice using properties of operations.

Prior Knowledge and Experiences

Student familiarity with plotting points on a graph and operations with integers will provide a strong foundation for this unit. This unit also builds on students understanding of proportional relationships developed in seventh grade and extended to similar triangles in eighth grade including the concept of the constant of proportionality. Students will also extend their prior knowledge with variables in relation to linear contexts.

Learning Experience # 1	
The teacher may provide a variety of real-world contexts so that students can begin to explore multiple representations of proportional relationships. *Enactive*: Students can explore several proportional relationships involving money (e.g., cost per item) with a partner and fake money (e.g., monopoly money) *Iconic*: Students can create a ratio table and coordinate graph for a variety of proportional relationships including those with a negative rate of change (e.g., digging swimming pool at a rate of 2ft per hour). Students can explain the meaning of the unit rate (rate-of-change) in the table and graph to a partner. *Symbolic*: Students can write an equation for each proportional relationship and describe the connection between the ratio table, coordinate graph and equation.	
Teacher Notes:	Students may struggle to explain the rate-of-change (unit rate) in each of the representations of the proportional relationship, it is important in this early learning experience to take the time for students to gain comfort with the meaning of the unit rate. This learning experience provides students with opportunities to compare proportional relationships represented in multiple ways by comparing the tables, graphs, and equations from a variety of contexts. Students can notice that all the graphs are increasing at different rates. Students often have a misconception that graphs with rates of change between 0 and 1 are decreasing. It is important to include graphs with fractional unit rates and to discuss these graphs compared to graphs with negative unit rates. Teachers may also want to remind students of the concepts of independent and dependent variables, these ideas were developed in sixth grade.
Generalization Connection(s):	Linear equations use y=mx to model proportional relationships and y=mx+b to model relationships in terms of a rate of change and initial value Rate of change and intercepts exist in all representations of linear relationships
Teacher Resources:	http://www.cpm.org/pdfs/state_supplements/Proportional_Relationships_Slope.pdf (contextualized examples of proportional relationships) http://learnzillion.com/lessonsets/275-graphing-interpreting-and-comparing-proportional-relationships (video lessons examining proportional relationships) http://www.illustrativemathematics.org/illustrations/57 (assessment tasks comparing speeds in graphs and equations) https://www.desmos.com/calculator (online graphing tool students can use to explore a variety of proportional relationships)
Student Resources:	http://www.ixl.com/promo?partner=google&campaign=1233&redirect=%2Fmath%2Fstandards%2Fcommon-core%2Fgrade-8&adGroup=8th+Grade&gclid=CLSnyqTanL0CFUVgMgodyy0A5A (practice questions on proportional relationships)
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: How is the rate of change represented in a table, graph and equation of a proportional relationship? Why is the table of a proportional relationship a ratio table? What does the rate of change mean in each proportional relationship you explored? How can you create one representation of a proportional relationship from another representation? What does it mean for a graph to be increasing? How can you determine from a table or equation if a graph will be increasing? What does a decreasing graph look like? What does the point (0,0) mean in each of the graphs you created? Why does each graph include this point?

Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide templates for creating the tables and graphs of proportional relationships (http://www.docstoc.com/docs/15477213/Function-Table-Graph-Template)	Students can make tables and graphs using templates
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://stricklerwms.weebly.com/8ee5-graph-and-compare-proportional-relationships.html (examples real-world examples of proportional relationships) http://www.cobbk12.org/Cheathamhill/LFS%20Update/Graphic%20Organizers.htm (graphic organizer templates)	Students can compare and contrast two of their own real-world proportional relationships using a graphic organizer
Key Knowledge and Skills:	• Graph proportional relationships, interpreting the unit rate as the slope of the graph • Compare two different proportional relationships represented in different ways • Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values	
Critical Language:	Rise, run, slope, vertical, horizontal, Interpret, graph, relationship, rate of change, proportional, representation, unit rate, ratio table, ratio, equation	

Learning Experience # 2

The teacher may demonstrate how to measure the rise and run of a variety of staircases so that students can explore the concept of steepness (i.e., slope).

Enactive: Students can examine several different staircases and predict which staircase is steepest. Students can then measure the rise and run of steps on a staircase for several different staircases.

Iconic: Students can draw scale models of each staircase and label the rise and run. Students can create a ratio table showing the rise and run for several different quantities of stairs for each staircase.

Symbolic: Students can graph the ratio table for each staircase on the same coordinate grid and compare the graphs to the drawings of each staircase and to each other. Students can then order the staircases by steepness and compare this new ordering to their initial predictions.

Teacher Notes:	This learning experience focuses on the concept of steepness to further develop students' understanding of slope. Usually stairs are built with a consistent rise and run but this is not always the case. If it turns out the slopes of different steps are different and its more than a little measurement error it might be good to discuss with students the impact of those differences. Often a step of different steepness in a staircase can lead to a fumble by someone new with the staircase. If several staircases are not available for students to measure, staircases can be built from a variety of blocks (e.g., legos, multi-linking cubes).
Generalization Connection(s):	Right triangles created between any two distinct points on a non-vertical line in a coordinate plane form similar triangle with a constant of proportionality equivalent to the slope of the line Rate of change and intercepts exist in all representations of linear relationships

Teacher Resources:	http://www.ez-stairs.com/widgets/stair_calculator/index.html (diagram showing how to measure rise and run of a staircase) https://moodle.centerville.k12.oh.us/mod/resource/view.php?id=18407 (lesson for exploring staircases and slope)	
Student Resources:	http://www.ixl.com/promo?partner=google&campaign=1233&redirect=%2Fmath%2Fstandards%2Fcommon-core%2Fgrade-8&adGroup=8th+Grade&gclid=CLSnyqTanL0CFUVgMgodyy0A5A (practice questions on proportional relationships)	
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: How do you determine the steepness of a staircase? What is slope? How does slope relate to staircases? Why are rise and run measurements used to determine the steepness of a staircase? Does the slope change if you measure the rise and run of multiple steps versus one step? Why or why not? When building stairs what do you need to consider? Why do we always state the rise and then the run when we are investigating slope?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://fawnnguyen.com/2012/05/03/20120503.aspx (worksheet with drawings of staircases)	Students can determine the steepness of staircases provided on a worksheet
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://www.ez-stairs.com/widgets/stair_calculator/index.html (calculator for determining the rise and run of stairs)	Students can experiment with a staircase calculator and explain how the calculator determines the rise and run Students can create a presentation on local building codes for the rise and run of staircases and compare them to the rise and run of wheelchair ramps
Key Knowledge and Skills:	• Graph proportional relationships, interpreting the unit rate as the slope of the graph • Compare two different proportional relationships represented in different ways	
Critical Language:	Rise, run, slope, vertical, horizontal, Interpret, graph, relationship, rate of change, proportional, representation, unit rate, ratio table, ratio	

Learning Experience # 3

The teacher may provide a variety of right triangles so that students can investigate the relationship between slope and similar right triangles.

Enactive: Students can cut out a variety of right triangles and order the triangles by steepness without making any measurements.

Iconic: Students can graph each triangle on a coordinate grid by lining up one leg of the right triangle along the x-axis starting at the origin and graphing the location of the vertex formed by the second leg and the hypotenuse. Students can then explore which triangles form a line through the origin.

Symbolic: Students can prove that triangles which form the same line are similar because they have the same constant of proportionality (slope) by using a ratio table based on the two legs of the triangles (rise and run) and vice versa (i.e., triangles not on the same line are not similar and have different constants of proportionality).

Teacher Notes:	If students are not familiar with similarity it may be necessary to do a mini-lesson on the idea of similarity prior to this learning experience. The goal of this learning experience is for students to explore different ways of visualizing steepness through the lens of similar triangles (e.g., graphs and ratio tables). Students should be asked to calculate slopes of lines throughout the remainder of the unit to build fluency with this skill. Students often struggle when calculating slope determining whether the ratio is rise to run or run to rise. This needs to be explicitly discussed and continually reinforced with students. They also struggle with subtracting two points on a line in the same order and often find a slope with the wrong sign. The teacher may want to show students an example of a graph and an incorrect calculation of slope and ask students to critique the solution. This can help students to understand the difference between positive and negative slope and ensure they calculate it correctly.	
Generalization Connection(s):	Right triangles created between any two distinct points on a non-vertical line in a coordinate plane form similar triangle with a constant of proportionality equivalent to the slope of the line Rate of change and intercepts exist in all representations of linear relationships	
Teacher Resources:	http://learnzillion.com/lessonsets/274-use-similar-triangles-to-explain-why-the-slope-m-is-the-same-between-two-points-on-a-nonvvertical-line-in-the-coordinate-plane (videos explaining why similar right triangles form the same line) http://www.illustrativemathematics.org/illustrations/1537 (assessment tasks about similar right triangles and slope) http://learnzillion.com/lessons/1357-prove-two-figures-are-similar-after-a-dilation (video about similarity) http://imathworksheets.com/wp-content/uploads/2012/09/slope-of-a-line-1.pdf (worksheet to practice slope using formula) http://www.youtube.com/watch?v=FmIhlc1bJuA&list=PLuXdoIfUT7tORhBjIiml-46yKpubQ3Gzi (song about slope: rise over run)	
Student Resources:	http://www.shmoop.com/common-core-standards/ccss-8-ee-6.html (explanation and questions about similar right triangles and slope)	
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: What do you notice about the ratios of the horizontal and vertical sides between similar triangles that you graphed on the coordinate plane? How does the constant of proportionality relate to the slope of a line? How can you calculate slope by creating a right triangle from any two distinct points on a non-vertical line? Why are the ratios of corresponding legs of two triangles, created on the same non-vertical line, proportional?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide pre-cut triangles made from card stock for some students. http://www.mathsisfun.com/geometry/triangles-similar.html (notes about similar triangles)	Students can compare the steepness of triangles with a partner for triangles whose legs are a whole number of units long Students can answer questions about why two triangles are similar by using notes about similar triangles
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can use the Pythagorean theorem to determine additional similar triangles and triangles with steepness between the provided triangles

Key Knowledge and Skills:	• Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane • Graph proportional relationships, interpreting the unit rate as the slope of the graph • Compare two different proportional relationships represented in different ways
Critical Language:	Rise, run, slope, vertical, horizontal, Interpret, graph, relationship, rate of change, proportional, representation, unit rate, ratio table, ratio, constant of proportionality, similar, right triangle, hypotenuse, legs

Learning Experience # 4	
The teacher may provide real-world contexts so that the students can extend the concept of proportional relationships (i.e., y=mx) to the more general linear relationship (i.e., y=mx+b). *Enactive*: Students can revisit some of proportional relationships from learning experience one and change the scenarios to involve a y-intercept other than the origin. *Iconic*: Students can create a table and coordinate graph for the linear relationships they created in the enactive part of the lesson including those with a negative slope and/or negative y-intercept. Students can explain the meaning the y-intercept in the table and graph to a partner and why the slope in the proportional relationship is the same as the linear relationship. *Symbolic*: Students can write an equation for each linear relationship and describe the connection between the table, coordinate graph and equation.	
Teacher Notes:	The teacher may approach this through the idea of a vertical translation. Students can begin with their graphs from the first learning experience and discuss how those graphs will change when the y-intercept changes. This is the first time students are encountering a linear relationship that is not proportional. It is important for students to recognize that an x, y table for this type of linear relationship is not a ratio table and proportional reasoning does not apply. Students will also need to explore how to calculate the slope from the x, y table and graph when given a linear rather than proportional relationship. Students sometimes struggle with the difference between y-intercepts versus x-intercepts. Students should be encouraged to explain why the letter b in equations of the form y=mx+b is the y-intercept and not the x-intercept.
Generalization Connection(s):	Linear equations use y=mx to model proportional relationships and y=mx+b to model relationships in terms of a rate of change and initial value Rate of change and intercepts exist in all representations of linear relationships
Teacher Resources:	http://www.cpm.org/pdfs/state_supplements/Proportional_Relationships_Slope.pdf (contextualized examples of proportional relationships) http://www.cimt.plymouth.ac.uk/projects/mepres/book9/bk9i5/bk9_5i6.html (examples real-world examples of linear relationships) http://map.mathshell.org/materials/lessons.php?taskid=211&subpage=concept (lesson plan that scaffolds the creation of linear equations from a context) http://www.bigideasmath.com/protected/content/ipe_na/grade%208/03/g8_03_04.pdf (practice with linear relationships) http://www.kutasoftware.com/FreeWorksheets/PreAlgWorksheets/Slope.pdf (worksheets to practice slope and y-intercept)
Student Resources:	http://www.math-play.com/slope-intercept-game.html (game to practice slope and intercept) http://www.algebralab.org/practice/practice.aspx?file=Algebra1_5-1.xml (practice writing equations in the form y=mx+b)

Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: When looking at a table or graph, how can you determine when the equation of a line will be in the form y=mx versus y=mx+b? How does changing the y-intercept value change the structure of your equation? What do m and b represent in a linear relationship? How can the rate of change and y-intercept be determined from a graph, table and equation? How can you write a linear equation from a graph, table or context?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may provide templates for creating the tables and graphs (http://www.docstoc.com/docs/15477213/Function-Table-Graph-Template)	Students can create the tables and graphs for each scenario using templates provided by the teacher
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://www.cobbk12.org/Cheathamhill/LFS%20Update/Graphic%20Organizers.htm (graphic organizer templates)	Students can compare and contrast linear and proportional relationships
Key Knowledge and Skills:	• Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values • Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. • Construct a function to model a linear relationship between two quantities and determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph • Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept	
Critical Language:	Slope, rise, run, intercept, graph, linear, proportional, table, equation, y=mx, y=mx+b	

Learning Experience # 5	
The teacher may demonstrate how to solve one-variable linear equations so that the students can explore the relationship between inverse operations and maintaining the balance of equations. *Enactive*: Students can use a pan balance and manipulations to solve one-variable linear equations. *Iconic*: Students can use the image of a pan balance to represent and solve one-variable linear equations. *Symbolic*: Students can solve one-variable linear equations using inverse operations and check the solution by substitution.	
Teacher Notes:	Teachers may want to include word problems in the examples so that students can practice solving equations in a context. A context provides students opportunities to review their answers in relation to the context of the problem to determine if the solution is reasonable.
Generalization Connection(s):	The creation of equivalent algebraic equations provides the necessary foundations for solving linear equations in one variable

Teacher Resources:	http://learnzillion.com/lessons/1009-solve-linear-equations-by-combining-like-terms (videos showing how to solve linear equations)	
Student Resources:	http://www.youtube.com/watch?v=63IkBH4kXzE (direct instruction example of solving multi-step equations) http://www.mathplayground.com/AlgebraEquations.html (balance scale problems for one and two-step equations) http://illuminations.nctm.org/Activity.aspx?id=3530 (illuminations pan balance for numbers) http://illuminations.nctm.org/Activity.aspx?id=3529 (illuminations pan balance for expressions and graphs)	
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: How are equivalent expressions used in the process of solving one-variable linear equations? How does the context of the problem relate to the reasonableness of a solution? How can inverse operations be used to create equivalent expressions? How is an equal sign similar to a fulcrum?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	The teacher may color code the constants, variables, and coefficients in an equation to scaffold the solution process for students	Students can solve one-variable equations with color-coded scaffolding
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://learnzillion.com/lessons/1009-solve-linear-equations-by-combining-like-terms (video showing how to solve one-variable equations)	Students can create a presentation (e.g. Prezi) that explains to other students how to solve one-variable linear equations
Key Knowledge and Skills:	• Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms	
Critical Language:	Coefficient, constant, variable, solution, operation, order of operations, distributive property, rational	

Learning Experience # 6	
The teacher may provide parameters for building equations (e.g., show me an equation with both subtraction and division where x=6) so that students can explore the relationship between building and solving equations. *Iconic*: Students can show a variety of operations operating on a variable using an “algebra machine” (i.e., flow chart) (see teacher resource). *Symbolic*: Students can transform their algebra machine into an equation with formal notation. Students can then solve a partner’s equation.	
Teacher Notes:	This learning experience provides students with opportunities to construct equations and then deconstruct the equation of a partner. It reinforces the previous learning experience by allowing students to develop complex equations and practice solving them. By first constructing the equation, students build confidence in how an equation is built and therefore how to unwind or deconstruct it to find the original value of the unknown. Students will need support in using formal notation, in particular they may need to be reminded the role of parentheses and the use of a fraction bar to represent division. Materials for the lesson can be found at the Shell Centre website including sample student response with errors for students to analyze and critique (http://map.mathshell.org/materials/lessons.php?taskid=487&subpage=concept).

Generalization Connection(s):	The creation of equivalent algebraic equations provides the necessary foundations for solving linear equations in one variable	
Teacher Resources:	http://map.mathshell.org/materials/lessons.php?taskid=487&subpage=concept (formative assessment lesson with pre and post assessment tasks, templates, and sample students for students to analyze)	
Student Resources:	http://www.fi.uu.nl/wisweb/en/ (applets called solving equations or linear equations are designed to practice solving one variable equations)	
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: How can you check your solution for the variable is correct? How can you rewrite an expression to show the whole expression multiplied by 3? How do you rewrite an expression to show the whole expression divided by 3? What are two different equations for which x=6? How do you know? How did your partner's solution strategy to find x compare to your strategy for building the equation? Are the operations you use to solve an equation the exact reverse of those used to solve it? Why or why not?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://map.mathshell.org/materials/lessons.php?taskid=487&subpage=concept (lesson plan with pre-made algebra machines and partially completed solutions)	Students can create equations from already created algebra machines Students can solve equations from partially completed solutions
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	N/A	Students can create equations with variables on both sides of the equation for their partner to solve
Key Knowledge and Skills:	• Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms	
Critical Language:	Coefficient, constant, variable, solution, operation, order of operations, distributive property, rational, parentheses, equal, balance, reverse, inverse, expression, equation	

Learning Experience # 7	
The teacher may provide a variety of one-variable linear equations so that students can begin to explore the differences between equations with only one, infinite, or no solutions. *Iconic*: Students can sort a variety of one-variable equations into categories of sometimes true, always true or never true. *Symbolic*: Students can explain how to create an equation that has one solution, infinite solutions and no solutions based on the simplest form of each type of equation, $x = a$, $a = a$, or $a = b$.	
Teacher Notes:	Students struggle sometimes to recognize when an equation is always true there are infinite solutions. Students can be encouraged to try a variety of numbers beyond whole number to help reinforce this idea, although not a proof this trial and error can support students' developing understanding.

Generalization Connection(s):	The creation of equivalent algebraic equations provides the necessary foundations for solving linear equations in one variable	
Teacher Resources:	http://map.mathshell.org/materials/download.php?fileid=1286 (lesson plan for exploring when equations have one, infinite or no solutions)	
Student Resources:	http://learnzillion.com/lessons/152-solve-equations-with-infinite-solutions (video about equations with infinite solutions) http://www.shmoop.com/common-core-standards/ccss-8-ee-7a.html (explanation and practice questions about equations with one, infinite and no solutions)	
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: What math can you use to show that a value of x satisfies an equation? How do you know if an equation will have infinite or no solutions? Can you think of an equation that is sometimes true with more than one solution? Why or why not? How many solutions does an equation need to be true? What does it mean for an equation to always be true? What does it mean for an equation to never be true?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://map.mathshell.org/materials/download.php?fileid=1286 (equation sorting cards)	Students can sort fewer cards into the categories of always, sometimes and never by working with a partner.
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://map.mathshell.org/materials/download.php?fileid=1286 (equation sorting cards)	Students can create additional cards to sort into always, sometimes and never and have a partner sort their cards.
Key Knowledge and Skills:	• Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms • Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions; show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers)	
Critical Language:	Coefficient, constant, variable, solution, operation, order of operations, distributive property, rational	

Learning Experience # 8	
The teacher may use matching activities (consisting of graphs, equations and contexts) so that students can explore rates of change and y-intercepts in multiple representations from multiple situations.	
Teacher Notes:	This learning experience is the culmination of the skills and understandings developed throughout the unit. Students can be provided with matching cards (see teacher resource) of graphs, equations, and contexts. Some of the representations needed to complete the matching activity are missing to allow students practice in creating graphs and equations from other representations.

Generalization Connection(s):	Linear equations use $y = mx$ to model proportional relationships and $y = mx + b$ to model relationships in terms of a rate of change and initial value Rate of change and intercepts exist in all representations of linear relationships	
Teacher Resources:	http://map.mathshell.org/materials/lessons.php?taskid=440&subpage=concept (lesson with card sort of graphs, equations, and contexts) http://nimitz9livingston.pbworks.com/f/Obj+3+Lesson+21+Wkbk.pdf (additional practice interpreting graphs)	
Student Resources:	http://www.mathwarehouse.com/algebra/linear_equation/slope-intercept-form.php (detailed explanation and practice of slope and y-intercepts) http://www.ixl.com/math/grade-8 (linear functions and slope extra practice) http://www.youtube.com/watch?v=03kVCQAZgA0 (direct instruction for interpreting slope and intercept)	
Assessment:	Students mastering the concept and skills of this lesson should be able to answer questions such as: How can you determine if an equation matches a graph and vice versa? Why is it important to pay attention to the scale on the vertical and horizontal parts of a graph? How can you tell by a graph if a slope is negative? What will a graph look like if an equation has a negative slope? What does the slope mean in each situation? Where is the y-intercept located on a graph and in an equation? What does the y-intercept mean in this situation?	
Differentiation: (Multiple means for students to access content and multiple modes for student to express understanding.)	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://map.mathshell.org/materials/lessons.php?taskid=440&subpage=concept (lesson with matching cards)	Students can be provided few cards to match such as match only the positive slope graphs and equations and then match the negative slope graphs and equations
Extensions for depth and complexity:	**Access** (Resources and/or Process)	**Expression** (Products and/or Performance)
	http://map.mathshell.org/materials/lessons.php?taskid=440&subpage=concept (lesson with matching cards)	Students can match the graph with the context and then write the equation for each graph rather than being given the equation matching cards
Key Knowledge and Skills:	• Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values • Derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. • Construct a function to model a linear relationship between two quantities and determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph • Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept	
Critical Language:	Equation, graph, y-intercept, positive slope, negative slope, rate of change, initial value, horizontal, vertical	